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Stress–singularity analysis in space junctions of thin plates

S. E. MIKHAILOV[1] and I. V. NAMESTNIKOVA[2]
[1]*Wessex Institute of Technology, Ashurst Lodge, Ashurst, Southampton SO40 7AA, U.K.*
e-mail: mik@wessex.ac.uk
[2]*Moscow State Academy of Instrumentation and Informatics, Dept. of Higher Math., 107076 Moscow, Stromynka 20, Russia*

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Abstract. The stress singularity in space junctions of thin linearly elastic isotropic plate elements with zero bending rigidities is investigated. The problem for an intersection of infinite wedge-shaped elements is considered first and the solution for each element, being in the plane stress state, is represented in terms of holomorphic functions (Kolosov–Muskhelishvili complex potentials) in some weighted Hardy-type classes. After application of the Mellin transform with respect to radius, the problem is reduced to a system of linear algebraic equations. By use of the residue calculus during the inverse Mellin transform, the stress asymptotics at the wedge apex is obtained. Then the asymptotic representation is extended to intersections of finite plate elements. Some numerical results are presented for a dependence of stress singularity powers on the junction geometry and on membrane rigidities of plate elements.

Key words: elasticity, stress singularity, stress asymptotics, space junction, transmission conditions.

1. Introduction

Three-dimensional structure junctions consisting of plane plate elements are generally used in engineering. The points, where plate intersection lines $\Gamma^{(l)}$ meet each other and/or free edges of plates, hereafter being referred to as singular points, are often stress concentrators. Examples of such points for V-shaped and T-shaped junctions of plates are presented in Figure 1.

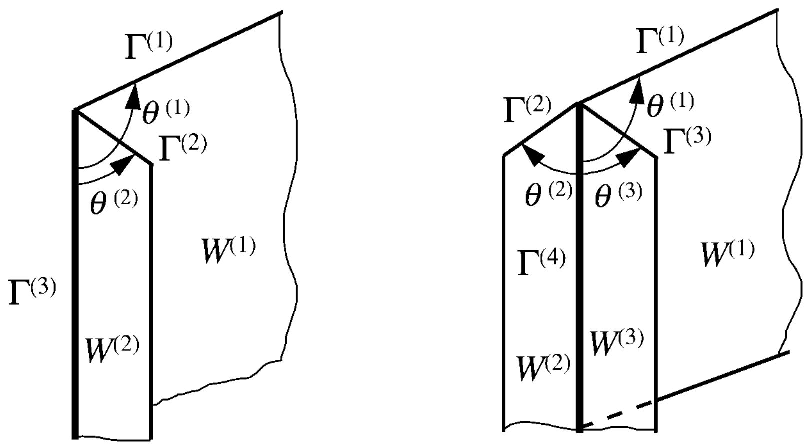

Figure 1. Schemes of V- and T-shaped junctions

Numerous studies are devoted to the investigation of elastic theory solution behaviour at singular points in two-dimensional bodies and at singular lines in three-dimensional bodies. The solution asymptotics for general elliptic boundary-value problems are studied in [1–3] (see also [4]). Asymptotics in some problems of solid mechanics with singular points are considered in [5–15]. In [16, Section 5], the singular behaviour of solutions is investigated for space junctions of plate elements with infinite membrane rigidities and finite bending rigidities.

Another limiting case, namely a space junction of plate elements with finite membrane rigidities and zero bending rigidities will be considered in this paper. Such neglect of the bending rigidities may be done for sufficiently thin plates and is rather popular in engineering computations. It delivers seemingly intermediate solution asymptotics valid at large distances from the singular point in comparison with the plate thicknesses. Under this assumption the plate elements of a junction will be in the plane stress state and have some special transmission conditions along the joint lines.

In Section 2 of this study, the problem statement is given for a three-dimensional intersection of infinite wedge-formed plates with zero bending rigidities, where each plate is in the plane stress state, and corresponding transmission conditions are deduced. (Note that the transmission conditions for intersecting Kirchhoff or Reissner–Mindlin plates with finite rigidities is thoroughly analysed in [17].) The stress singularity analysis for such problems can be executed in different function classes. For example, in [1–4], [16] the solution asymptotics for some problems are analysed in the weighted or usual Sobolev spaces. We will work in the Hardy-type weighted classes of holomorphic functions (for the Kolosov–Muskhelishvili potentials) introduced in [18] and being sufficiently convenient for an application of the integral Mellin transform. In Section 3, after the application of the Mellin transform to the representation of a general solution in terms of the Kolosov–Muskhelishvili complex potentials, the problem is reduced to a system of linear algebraic equations with a parameter. The employment of the residue calculus and some properties of the Hardy-type functions in the inverse Mellin transform allow us to obtain in Section 4 an asymptotic representation for the problem solution. By means of the cut-off function techniques, the asymptotic representations can be extended to intersections of plates of arbitrary form. In Section 5, stress singularity powers are numerically evaluated at singular points for some standard junction geometries. Critical junction values (angles, rigidities), separating the parameter zones with and without the stress singularities, are analysed too.

2. Problem statement, boundary and transmission conditions

Let $\{x_\alpha\}$, $\alpha = 1 \ldots 3$, be global Cartesian coordinates. Hereafter, unless otherwise stated, the summation in repeating subscripts (excluding the subscripts ρ and θ) is supposed from 1 to 3; superscript summation is not done unless it is pointed out explicitly. Plate elements are considered below as two-dimensional plane elastic objects and the term thickness H is used only to calculate plate tractions (per unit length) $H\sigma_{\alpha\beta}$ from plate stresses $\sigma_{\alpha\beta}$.

Let us consider M plates intersecting along a joint line Γ. Let k_α be the unit tangent to Γ. For each plate $W^{(m)}$ $(m = 1, \ldots, M)$ adjoining Γ, we denote by $n_\alpha^{(m)}$ the normal to Γ lying in $W^{(m)}$ plane and being external to $W^{(m)}$. Let $H^{(m)}$ be the plate thickness. Then the sum of the boundary tractions transmitted from each plate to Γ must be equal to a prescribed load g_j

$$\sum_{m=1}^{M} H^{(m)}\sigma^{(m)}_{j\beta}n^{(m)}_{\beta} = g_j, \quad j = 1, \dots, 3. \tag{2.1}$$

Moreover the displacements $u^{(m)}_j$ of each plate at Γ must be equal to one and the same unknown function U_j

$$u^{(m)}_j = U_j, \quad j = 1, \dots, 3, \quad m = 1, \dots, M. \tag{2.2}$$

Taking into account that the plates do not resist the bending moments and, hence, the transverse tractions, we have that only membrane stresses are involved in (2.1). Then these equations can be rewritten in the form

$$\sum_{m=1}^{M} H^{(m)}[\sigma^{(m)}_{\alpha\beta}k_\alpha n^{(m)}_{\beta}k_j + \sigma^{(m)}_{\alpha\beta}n^{(m)}_{\alpha}n^{(m)}_{\beta}n^{(m)}_{j}] = g_j, \quad j = 1, \dots, 3. \tag{2.3}$$

From Equations (2.2) we obtain the equations for the membrane displacements

$$u^{(m)}_j k_j = U_j k_j, \quad u^{(m)}_j n^{(m)}_j = U_j n^{(m)}_j, \quad m = 1, \dots, M. \tag{2.4}$$

Let us assume that not all plates lie in one plane (not all $n^{(m)}_j$ are parallel). Then (2.3) gives three independent equations with respect to $2M$ boundary tractions $\sigma^{(m)}_{\alpha\beta}k_\alpha n^{(m)}_{\beta}$, $\sigma^{(m)}_{\alpha\beta}n^{(m)}_{\alpha}n^{(m)}_{\beta}$. This can be verified by projection of (2.3) onto the tangent k_j and onto any two not coincident normals $n^{(m_1)}_j$, $n^{(m_2)}_j$. Excluding three auxiliary functions U_j from $2M$ Equations (2.4), we get then from (2.3), (2.4) $2M$ independent boundary conditions for $4M$ values: the boundary membrane tractions $\sigma^{(m)}_{\alpha\beta}k_\alpha n^{(m)}_{\beta}$, $\sigma^{(m)}_{\alpha\beta}n^{(m)}_{\alpha}n^{(m)}_{\beta}$ and the displacements $u^{(m)}_j k_j$, $u^{(m)}_j n^{(m)}_j$.

If all the plates lie in one plane with a normal ζ_j, *i.e.*, $n^{(m)}_j = \pm n^{(1)}_j$, $m = 1, \dots, M$, then we have from (2.3) only two independent conditions

$$\sum_{m=1}^{M} H^{(m)}\sigma^{(m)}_{\alpha\beta}k_\alpha n^{(m)}_{\beta} = g_j k_j, \qquad \sum_{m=1}^{M} H^{(m)}\sigma^{(m)}_{\alpha\beta}n^{(1)}_{\alpha}n^{(m)}_{\beta} = g_j n^{(1)}_j. \tag{2.5}$$

The third relation, which can be obtained from (2.3), gives the solvability condition imposed on the prescribed tractions: $g_j\zeta_j = 0$. In this case, $2M$ relations (2.4) involve only two auxiliary functions $U_j k_j$ and $U_j n^{(1)}_j$; excluding them, we obtain from (2.5) and (2.4) again $2M$ independent conditions for $4M$ boundary membrane tractions and displacements.

When Γ is a plate edge not in contact with other plates, we will also call Γ a joint line for $M = 1$. If some tractions g_j are prescribed on it, then we have there two boundary conditions (2.5) where the sum sign must be dropped, that is, $M = 1$ must be substituted. If some displacements f_i are prescribed instead of tractions on this edge, then we have two boundary conditions (2.4), but for known functions $U_j = f_j$ and for $M = 1$.

Thus, if M plates contact along a joint line Γ, $M \geqslant 1$, then there are generally $2M$ independent transmission/boundary conditions on this line for $4M$ boundary membrane tractions and displacements. This means that each plate edge generates two transmission/boundary conditions on Γ. Below the transmission conditions will also be called boundary conditions.

Let L joint lines $\Gamma^{(l)} \quad (l = 1, \dots, L)$ of M^* wedge-shaped plates $W^{(m)}$, $m = 1, \dots, M^*$, intersect in the point $x_j = 0$. It is simple to see that $1 \leqslant L \leqslant 2M^*$. Since each wedge-shaped plate has two edges, there are $4M^*$ boundary conditions on L joint lines $\Gamma^{(l)}$ for $8M^*$ boundary membrane tractions and displacements.

Let us introduce the local Cartesian $\{y_j\}$, $j = 1, 2$, and the polar (ρ, θ) coordinate systems in the plane of each plate with the origin in the corner point. Then the m-th plate is a wedge $W^{(m)} := W(0, \theta^{(m)}) : \quad (\rho, \theta) \in (0, \infty) \times (0, \theta^{(m)})$, whose plane stress state is described by the plane elasticity equations

$$\sigma^{(m)}_{j\alpha,\alpha} = 0, \qquad \sigma^{(m)}_{j\alpha} = \Lambda^{(m)}_{*} u^{(m)}_{\beta,\beta} \delta_{j\alpha} + \mu^{(m)} (u^{(m)}_{j,\alpha} + u^{(m)}_{\alpha,j}), \quad (j, \alpha, \beta = 1, 2). \tag{2.6}$$

Here $\Lambda_* := 2\Lambda\mu/(\Lambda + 2\mu)$; Λ and μ are the Lamé constants. The corresponding boundary conditions are prescribed at both edges of the wedge. For example, if some tractions $g_\rho(\rho)$, $g_\theta(\rho)$ are prescribed at the edge $\theta = \theta^{(m)}$, then the boundary conditions there have the form

$$\sigma_{\theta\theta}(\rho, \theta^{(m)}) = g_\theta(\rho), \qquad \sigma_{\rho\theta}(\rho, \theta^{(m)}) = g_\rho(\rho). \tag{2.7}$$

If some displacements $f_\rho(\rho)$, $f_\theta(\rho)$ are prescribed there, then we have

$$u_\theta(\rho, \theta^{(m)}) = f_\theta(\rho), \qquad u_\rho(\rho, \theta^{(m)}) = f_\rho(\rho). \tag{2.8}$$

If, instead, the mth plate contacts through this edge with the other $M-1$ plates, the local forms of the corresponding transmission conditions are obtained from (2.3)–(2.5), by expressing the displacements and the stresses there in local polar coordinate systems.

For example, the boundary conditions for a V-junction (Figure 1) have the form

$$\begin{aligned}
&\Gamma^{(1)}: \quad \sigma^{(1)}_{\theta\theta}(\rho, \theta^{(1)}) = g^{(1)}_\theta(\rho), \qquad \sigma^{(1)}_{\rho\theta}(\rho, \theta^{(1)}) = g^{(1)}_\rho(\rho),\\
&\Gamma^{(2)}: \quad \sigma^{(2)}_{\theta\theta}(\rho, \theta^{(2)}) = g^{(2)}_\theta(\rho), \qquad \sigma^{(2)}_{\rho\theta}(\rho, \theta^{(2)}) = g^{(2)}_\rho(\rho),\\
&\Gamma^{(3)}: \quad \sigma^{(1)}_{\theta\theta}(\rho, 0) = g^{(31)}_\theta(\rho), \qquad \sigma^{(2)}_{\theta\theta}(\rho, 0) = g^{(32)}_\theta(\rho),\\
&\qquad H^{(1)}\sigma^{(1)}_{\rho\theta}(\rho, 0) + H^{(2)}\sigma^{(2)}_{\rho\theta}(\rho, 0) = g^{(3)}_\rho(\rho), \qquad u^{(1)}_\rho(\rho, 0) - u^{(2)}_\rho(\rho, 0) = 0.
\end{aligned} \tag{2.9}$$

The general solution of system (2.6) can be written in terms of the complex Kolosov–Muskhelishvili potentials [19]. Particularly, the radial derivatives of the displacements $\partial u^{(m)}_\alpha/\partial\rho$, and the stresses $\sigma^{(m)}_{\alpha\beta}$ for mth plate in polar coordinates have the form

$$\begin{aligned}
\frac{\partial}{\partial\rho} u^{(m)}_\rho(\rho, \theta) &= \frac{1}{4\mu^{(m)}} \sum_{j=1}^{2} [(\kappa^{(m)} - 1)\Phi^{(m)}_j(z_j) - z_j \Phi^{(m)\prime}_j(z_j) - e^{2i\theta_j} \Psi^{(m)}_j(z_j)],\\
\frac{\partial}{\partial\rho} u^{(m)}_\theta(\rho, \theta) &= \frac{i}{4\mu^{(m)}} \sum_{j=1}^{2} (-1)^j [(\kappa^{(m)} + 1)\Phi^{(m)}_j(z_j) + z_j \Phi^{(m)\prime}_j(z_j) + e^{2i\theta_j} \Psi^{(m)}_j(z_j)],\\
\sigma^{(m)}_{\rho\rho}(\rho, \theta) &= \frac{1}{2} \sum_{j=1}^{2} [2\Phi^{(m)}_j(z_j) - z_j \Phi^{(m)\prime}_j(z_j) - e^{2i\theta_j} \Psi^{(m)}_j(z_j)],\\
\sigma^{(m)}_{\theta\theta}(\rho, \theta) &= \frac{1}{2} \sum_{j=1}^{2} [2\Phi^{(m)}_j(z_j) + z_j \Phi^{(m)\prime}_j(z_j) + e^{2i\theta_j} \Psi^{(m)}_j(z_j)],\\
\sigma^{(m)}_{\rho\theta}(\rho, \theta) &= \frac{i}{2} \sum_{j=1}^{2} (-1)^j [z_j \Phi^{(m)\prime}_j(z_j) + e^{2i\theta_j} \Psi^{(m)}_j(z_j)].
\end{aligned} \tag{2.10}$$

Here $\theta_1 := -\theta_2 := \theta$; $z_j(\rho,\theta) := \rho\exp(\mathrm{i}\theta_j) \in W_j^{(m)} := W(\theta_{j-}^{(m)}, \theta_{j+}^{(m)})$; $\theta_{1-}^{(m)} := \theta_{2+}^{(m)} := \theta_-^{(m)} = 0, \theta_{1+}^{(m)} := -\theta_{2-}^{(m)} := \theta_+^{(m)} = \theta^{(m)}$ and the general wedge notation $W(\theta_-, \theta_+) : \{(\rho,\theta) \in (0,\infty) \times (\theta_-, \theta_+)\}$ is used. In addition, $\kappa^{(m)} := (3 - \nu^{(m)})/(1 + \nu^{(m)})$, where $\nu^{(m)} := \Lambda^{(m)}/[2(\Lambda^{(m)} + \mu^{(m)})]$ are the Poisson ratios; $\Phi_j^{(m)}(z_j)$ and $\Psi_j^{(m)}(z_j)$ are analytical functions of the complex arguments z_j, the prime denote the derivative with respect to z_j.

Using expressions (2.10) at the boundaries $\theta = \theta_\mp^{(m)}$ ($\theta_j = \theta_{j\mp}^{(m)}$) of the wedges $W^{(m)}$, we substitute them in boundary conditions (2.3)–(2.5), (2.7)–(2.8). Thus, we arrive at the boundary-value problem for analytic functions: $4M^*$ holomorphic functions $\Phi_j^{(m)}(z_j)$, $\Psi_j^{(m)}(z_j)$ ($j = 1, 2$; $m = 1, \ldots, M^*$) must be determined from $4M^*$ boundary conditions. When speaking below about a solution, we mean a solution of this problem for analytic functions.

We define below some weighted Hardy-type function classes (considered in details in [18]), in which the solution will be looked for. Denote $z = \rho\exp(\mathrm{i}\theta)$. Let $H_2(\delta_0, \delta_\infty; W(\theta_-, \theta_+))$ be the class of functions $\Phi(z)$ holomorphic in a wedge $W(\theta_-, \theta_+)$ and such that $\sup_{\theta_- < \theta < \theta_+} \int_0^\infty |\Phi(\rho e^{\mathrm{i}\theta})|^2 \rho^{2\delta - 1}\, \mathrm{d}\rho < \infty$ for all $\delta \in (\delta_0, \delta_\infty)$.

Let us consider the following linear combinations of the complex potentials

$$\tilde{\Psi}_{j\mp}^{(m)}(z_j) := z_j \Phi_j^{(m)\prime}(z_j) + \exp(2\mathrm{i}\theta_{j\mp}^{(m)})\Psi_j^{(m)}(z_j).$$

In fact, only these combinations and the original potentials $\Phi_j^{(m)}(z_j)$ are involved in the representations (2.10) at the boundaries and, hence, in the boundary conditions (2.3)–(2.5), (2.7)–(2.8) (after the differentiation of the conditions for displacements with respect to ρ). We shall write that a pair of functions $\Phi_j^{(m)}$, $\Psi_j^{(m)}$ belongs to $\tilde{H}_2(\delta_0, \delta_\infty; W_j^{(m)})$, if $\Phi_j^{(m)} \in H_2(\delta_0, \delta_\infty; W_j^{(m)})$ and $\tilde{\Psi}_{j\mp}^{(m)} \in H_2(\delta_0, \delta_\infty; \tilde{W}_{j\mp}^{(m)})$, where

$$\tilde{W}_{j-}^{(m)} := W(\theta_{j-}^{(m)}, \tilde{\theta}), \tilde{W}_{j+}^{(m)} := W(\tilde{\theta}, \theta_{j+}^{(m)}), \quad \text{for any } \tilde{\theta} \in (\theta_{j-}^{(m)}, \theta_{j+}^{(m)}).$$

Let the prescribed boundary functions be

$$g_j(\rho),\ \mathrm{d}f_j(\rho)/\mathrm{d}\rho \in \hat{L}_2(\delta_g, 1), \quad \delta_g < 1, \tag{2.11}$$

where the class $\hat{L}_2(\delta_0, \delta_\infty)$ consists of the functions $g(\rho)$, such that $\int_0^\infty |g(\rho)|^2 \rho^{2\delta-1}\, \mathrm{d}\rho < \infty$, for all $\delta \in (\delta_0, \delta_\infty)$.

We look for, as a solution, the Kolosov–Muskhelishvili potentials

$$\Phi_j^{(m)}, \Psi_j^{(m)} \in \tilde{H}_2(\tilde{\delta}_0, 1; W_j^{(m)}), \tag{2.12}$$

for some $\tilde{\delta}_0 < 1$. The choice of $\tilde{H}_2$ for the generation of a solution is motivated by the following reasons. First, this class is sufficiently convenient for the application of the Mellin transform in complex variables and, moreover, the boundary values of the Mellin transforms are represented in terms of the Mellin transforms of the boundary values, for functions from this class (Lemma 1.16 in [18]). Second, the Kolosov–Muskhelishvili potentials from this class, and consequently, the stresses generated by them may have weak singularities at the wedge apex (Lemma 1.10 in [18]) and are square integrable over any finite two-dimensional part of $W^{(m)}$ (including also the singular point), *i.e.,* have a finite elastic energy there. In addition, if the potentials belong to $\tilde{H}_2(\tilde{\delta}_0, \tilde{\delta}_\infty; W_j^{(m)}) \subset \tilde{H}_2(\tilde{\delta}_0, 1; W_j^{(m)})$ for $\tilde{\delta}_0 < 1 < \tilde{\delta}_\infty$, then the elastic energy over the whole of $W^{(m)}$ is bounded (Lemmas 1.17–1.18 in [18]). Third, as will be seen below, the solution of the problem exists and is unique for $\Phi_j^{(m)}, \Psi_j^{(m)} \in \tilde{H}_2(\tilde{\delta}_0, 1; W_j^{(m)})$.

3. Problem solution

We will solve the problem using the Mellin transform of the complex potentials, which reduces the problem to an algebraic one. This idea seems to have been used first in [20]. Let $S(\delta_0, \delta_\infty)$ be the strip $\delta_0 < \Re\gamma < \delta_\infty$ in the complex γ-plane. (Hereafter, $\Re\gamma$ denotes the real part of γ.) It follows from Theorem 1.15 in [18] that the Mellin transforms with respect to complex variables exist for $\Phi_j^{(m)}, \Psi_j^{(m)} \in \tilde{H}_2(\tilde{\delta}_0, 1; W_j^{(m)})$ at any $\gamma \in S := S(\tilde{\delta}_0, 1)$,

$$\Phi_j^{(m)\vee}(\gamma) := \int_0^\infty \Phi_j^{(m)}(z) z^{\gamma-1} \mathrm{d}z, \qquad \Psi_j^{(m)\vee}(\gamma) := \int_0^\infty \Psi_j^{(m)}(z) z^{\gamma-1}\, \mathrm{d}z,$$

$$z \in W_j^{(m)}; \quad \tilde{\Psi}_{j\mp}^{(m)\vee}(\gamma) = -\gamma\, \Phi_j^{(m)\vee}(\gamma) + \exp(2\mathrm{i}\theta_{j\mp}^{(m)}) \Psi_j^{(m)\vee}(\gamma).$$

In addition, $\Phi_j^{(m)\vee}, \Psi_j^{(m)\vee} \in \tilde{H}_2^\vee(\theta_{j-}^{(m)}, \theta_{j+}^{(m)}; S)$, that is

$$\Phi_j^{(m)\vee} \in H_2^\vee(\theta_{j-}^{(m)}, \theta_{j+}^{(m)}; S) \quad \text{and} \quad \tilde{\Psi}_{j-}^{(m)\vee} \in H_2^\vee(\theta_{j-}^{(m)}, \tilde{\theta}; S), \quad \tilde{\Psi}_{j+}^{(m)\vee} \in H_2^\vee(\tilde{\theta}, \theta_{j+}^{(m)}; S)$$
$$\text{for any } \tilde{\theta} \in (\theta_{j-}^{(m)}, \theta_{j+}^{(m)}).$$

Here $H_2^\vee(\theta_-, \theta_+; S(\delta_0, \delta_\infty))$ is the class of functions $\Phi^\vee(\gamma)$ holomorphic in $S(\delta_0, \delta_\infty)$ and such that the norm $\sup_{\theta_- < \theta < \theta_+} [\int_{-\infty}^{\infty} |\Phi^\vee(\delta + \mathrm{i}\xi) \mathrm{e}^{\xi\theta}|^2 \mathrm{d}\xi]^{1/2}$ is uniformly bounded with respect to δ on any segment $[\delta_0', \delta_\infty'] \subset (\delta_0, \delta_\infty)$.

Let us denote by

$$\langle g \rangle(\gamma) := \int_0^\infty g(\rho) \rho^{\gamma-1}\, \mathrm{d}\rho$$

the Mellin transform with respect to a real variable. For a function $\Phi(z)$ from $H_2(\delta_0, \delta_\infty; W(\theta_-, \theta_+))$ there is the following property connecting the Mellin transforms with respect to the real variable ρ (over a line $\theta = \text{const.}$) and to the complex variable $z = \rho \mathrm{e}^{\mathrm{i}\theta}$ (Lemma 1.16 in [18]):

$$\langle \Phi \rangle(\gamma, \theta) = \mathrm{e}^{-\mathrm{i}\gamma\theta} \Phi^\vee(\gamma), \quad \gamma \in S(\delta_0, \delta_\infty), \;\; \theta \in [\theta_-, \theta_+].$$

Consequently, under condition (2.12) this property holds for $\Phi_j^{(m)}(z)$ in the segments $\theta_{j-}^{(m)} \leqslant \theta \leqslant \theta_{j+}^{(m)}$; for the combination $\tilde{\Psi}_{j-}^{(m)}(z_j)$ it holds only in the half interval $\theta_{j-}^{(m)} \leqslant \theta < \theta_{j+}^{(m)}$ and for $\tilde{\Psi}_{j+}^{(m)}(z_j)$ in $\theta_{j-}^{(m)} < \theta \leqslant \theta_{j+}^{(m)}$.

Having this in mind, we apply the Mellin transform with respect to ρ to the relations (2.10) and obtain the representations, coupling the Mellin transforms of the displacement derivatives and stresses with the Mellin transform of the complex potentials, which hold also on the boundaries $\theta = \theta_{1\mp}^{(m)}$ $(\theta_j = \theta_{j\mp}^{(m)})$

$$\left\langle \frac{\partial u_\rho^{(m)}}{\partial \rho} \right\rangle(\gamma,\theta) = \frac{1}{4\mu^{(m)}} \sum_{j=1}^{2} [(\kappa^{(m)} - 1 + \gamma)\, \mathrm{e}^{-\mathrm{i}\gamma\theta_j}\, \Phi_j^{(m)\vee}(\gamma) - \mathrm{e}^{\mathrm{i}(2-\gamma)\theta_j}\, \Psi_j^{(m)\vee}(\gamma)],$$

$$\left\langle \frac{\partial u_\theta^{(m)}}{\partial \rho} \right\rangle(\gamma,\theta) = \frac{\mathrm{i}}{4\mu^{(m)}} \sum_{j=1}^{2} (-1)^j [(\kappa^{(m)} + 1 - \gamma)\, \mathrm{e}^{-\mathrm{i}\gamma\theta_j}\, \Phi_j^{(m)\vee}(\gamma) + \mathrm{e}^{\mathrm{i}(2-\gamma)\theta_j}\, \Psi_j^{(m)\vee}(\gamma)],$$

$$\langle \sigma_{\rho\rho}^{(m)} \rangle(\gamma,\theta) = \frac{1}{2} \sum_{j=1}^{2} [(2+\gamma)\, \mathrm{e}^{-\mathrm{i}\gamma\theta_j}\, \Phi_j^{(m)\vee}(\gamma) - \mathrm{e}^{\mathrm{i}(2-\gamma)\theta_j}\, \Psi_j^{(m)\vee}(\gamma)],$$

$$\langle \sigma_{\theta\theta}^{(m)} \rangle(\gamma,\theta) = \frac{1}{2} \sum_{j=1}^{2} [(2-\gamma)\, \mathrm{e}^{-\mathrm{i}\gamma\theta_j}\, \Phi_j^{(m)\vee}(\gamma) + \mathrm{e}^{\mathrm{i}(2-\gamma)\theta_j}\, \Psi_j^{(m)\vee}(\gamma)],$$

$$\langle \sigma_{\rho\theta}^{(m)} \rangle(\gamma,\theta) = \frac{\mathrm{i}}{2} \sum_{j=1}^{2} (-1)^j [-\gamma\, \mathrm{e}^{-\mathrm{i}\gamma\theta_j}\, \Phi_j^{(m)\vee}(\gamma) + \mathrm{e}^{\mathrm{i}(2-\gamma)\theta_j}\, \Psi_j^{(m)\vee}(\gamma)]. \tag{3.1}$$

Applying the Mellin transform with respect to ρ to boundary conditions (2.3)–(2.5), (2.7)–(2.8) (differentiating preliminary in ρ the conditions for the displacements u) and using (3.1), we get a system of $4M^*$ linear algebraic equations to determine the $4M^*$ Mellin transforms $\Phi_j^{(m)\vee}(\gamma)$, $\Phi_j^{(m)\vee}(\gamma)$

$$\sum_{\beta=1}^{4M^*} B_{\alpha\beta}(\gamma)\mathcal{F}_\beta(\gamma) = \mathcal{G}_\alpha, \quad \alpha = 1, \ldots, 4M^*, \tag{3.2}$$

$$\{\mathcal{F}_\beta(\gamma)\} := \{\Phi_1^{(m)\vee}(\gamma), \Phi_2^{(m)\vee}(\gamma), \Psi_1^{(m)\vee}(\gamma), \Psi_2^{(m)\vee}(\gamma)\}, \quad m = 1, \ldots, M^*,$$

$$\{\mathcal{G}_\alpha(\gamma)\} := \{\langle \hat{g}_j \rangle(\gamma), \langle \mathrm{d}\hat{f}_j/\mathrm{d}\rho \rangle(\gamma)\},$$

where $\hat{g}_j$, $\hat{f}_j$ are obtained from the corresponding functions $g_j^{(l)}$, $f_j^{(l)}$ prescribed on the boundary.

For the V-junction (see boundary conditions (2.9)), for example, $M^* = 2$, the matrix $B_{\alpha\beta}$ has the form

$(2-\gamma)\,\mathrm{e}^{-\mathrm{i}\gamma\theta^{(1)}}$	$(2-\gamma)\,\mathrm{e}^{\mathrm{i}\gamma\theta^{(1)}}$	$\mathrm{e}^{\mathrm{i}(2-\gamma)\theta^{(1)}}$	$\mathrm{e}^{-\mathrm{i}(2-\gamma)\theta^{(1)}}$	0	0	0	0
$-\gamma\,\mathrm{e}^{-\mathrm{i}\gamma\theta^{(1)}}$	$\gamma\,\mathrm{e}^{\mathrm{i}\gamma\theta^{(1)}}$	$\mathrm{e}^{\mathrm{i}(2-\gamma)\theta^{(1)}}$	$-\mathrm{e}^{-\mathrm{i}(2-\gamma)\theta^{(1)}}$	0	0	0	0
0	0	0	0	$(2-\gamma)\,\mathrm{e}^{-\mathrm{i}\gamma\theta^{(2)}}$	$(2-\gamma)\,\mathrm{e}^{\mathrm{i}\gamma\theta^{(2)}}$	$\mathrm{e}^{\mathrm{i}(2-\gamma)\theta^{(2)}}$	$\mathrm{e}^{-\mathrm{i}(2-\gamma)\theta^{(2)}}$
0	0	0	0	$-\gamma\,\mathrm{e}^{-\mathrm{i}\gamma\theta^{(2)}}$	$\gamma\,\mathrm{e}^{\mathrm{i}\gamma\theta^{(2)}}$	$\mathrm{e}^{\mathrm{i}(2-\gamma)\theta^{(2)}}$	$-\mathrm{e}^{-\mathrm{i}(2-\gamma)\theta^{(2)}}$
$(2-\gamma)$	$(2-\gamma)$	1	1	0	0	0	0
0	0	0	0	$(2-\gamma)$	$(2-\gamma)$	1	1
$-H^{(1)}\gamma$	$H^{(1)}\gamma$	$H^{(1)}$	$-H^{(1)}$	$-H^{(2)}\gamma$	$H^{(2)}\gamma$	$H^{(2)}$	$-H^{(2)}$
$\frac{\kappa^{(1)}-1+\gamma}{\mu^{(1)}}$	$\frac{\kappa^{(1)}-1+\gamma}{\mu^{(1)}}$	$\frac{-1}{\mu^{(1)}}$	$\frac{-1}{\mu^{(1)}}$	$-\frac{\kappa^{(2)}-1+\gamma}{\mu^{(2)}}$	$-\frac{\kappa^{(2)}-1+\gamma}{\mu^{(2)}}$	$\frac{1}{\mu^{(2)}}$	$\frac{1}{\mu^{(2)}}$

$$\{\mathcal{F}_\beta(\gamma)\} = \{\Phi_1^{(1)\vee}, \Phi_2^{(1)\vee}, \Psi_1^{(1)\vee}, \Psi_2^{(1)\vee}, \Phi_1^{(2)\vee}, \Phi_2^{(2)\vee}, \Psi_1^{(2)\vee}, \Psi_2^{(2)\vee}\},$$
$$\{\mathcal{G}_\alpha(\gamma)\} = 2\{\langle g_\theta^{(1)}\rangle, i\langle g_\rho^{(1)}\rangle, \langle g_\theta^{(2)}\rangle, i\langle g_\rho^{(2)}\rangle, \langle g_\theta^{(31)}\rangle, \langle g_\theta^{(32)}\rangle, i\langle g_\rho^{(3)}\rangle, 0\}. \tag{3.3}$$

The solution of system (3.2) has the form

$$\mathcal{F}_\alpha(\gamma) = \sum_{\beta=1}^{4M^*} B_{\alpha\beta}^{-1}(\gamma)\mathcal{G}_\beta(\gamma), \quad B_{\alpha\beta}^{-1}(\gamma) = A_{\alpha\beta}(\gamma)/\Delta(\gamma), \tag{3.4}$$

where $\Delta(\gamma)$ is the determinant of the matrix $B_{\alpha\beta}(\gamma)$, $A_{\alpha\beta}(\gamma)$ is the transposed matrix of its algebraic complements and $B_{\alpha\beta}^{-1}(\gamma)$ is the inverse matrix to $B_{\alpha\beta}(\gamma)$.

Expressions (3.4) can be rewritten in terms of $\Phi_j^{(m)\vee}(\gamma)$, $\Psi_j^{(m)\vee}(\gamma)$, $\tilde{\Psi}_{j\mp}^{(m)\vee}(\gamma)$

$$\Phi_j^{(m)\vee} = \sum_{\beta=1}^{4M^*} B_{4m-4+j,\beta}^{-1}\mathcal{G}_\beta, \qquad \Psi_j^{(m)\vee} = \sum_{\beta=1}^{4M^*} B_{4m-2+j,\beta}^{-1}\mathcal{G}_\beta,$$
$$\tilde{\Psi}_{j\mp}^{(m)\vee} = \sum_{\beta=1}^{4M^*} \tilde{D}_{j\beta\mp}^{(m)}\mathcal{G}_\beta, \tag{3.5}$$
$$\tilde{D}_{j\beta\mp}^{(m)}(\gamma) := -\gamma B_{4m-4+j,\beta}^{-1}(\gamma) + \exp(2i\theta_{j\mp}^{(m)})B_{4m-2+j,\beta}^{-1}(\gamma).$$

One can see from (2.3)–(2.5), (2.7)–(2.8), (3.1) that $B_{\alpha\beta}(\gamma)$ and consequently $A_{\alpha\beta}(\gamma)$ and $\Delta(\gamma)$ are entire functions. Suppose, in addition, that $\Delta(\gamma)$ is not equal to zero identically (in the opposite case it would lead to the linearly dependent boundary conditions, what seems not to happen in correctly stated elasticity problems). Hence (see *e.g.* [21, Chapter V, Section 1]) $\Delta(\gamma)$ may have only isolated zeros γ_k of finite multiplicities N_k° in any finite part of γ-plane. By (3.4), consequently, $B_{\alpha\beta}^{-1}(\gamma)$ is a meromorphic function with poles of finite multiplicities in γ_k. Moreover, according to [22, Theorem 7.1], the matrix $B_{\alpha\beta}^{-1}(\gamma)$ has the form

$$B_{\alpha\beta}^{-1}(\gamma) = \sum_{n=1}^{N_k}\sum_{q=1}^{P_{kn}}(\gamma-\gamma_k)^{-q}\sum_{p=0}^{P_{kn}-q}\phi_{\alpha k}^{(np)}\chi_{\beta k}^{(n,P_{kn}-q-p)} + D_{\alpha\beta}(\gamma) \tag{3.6}$$

in the neighborhood of γ_k. Here N_k is the dimension of the eigenspace of the matrix $B_{\alpha\beta}(\gamma_k)$, $\phi_{\alpha k}^{(np)}$ and $\chi_{\beta k}^{(np)}$ $(n = 1, \ldots, N_k,\ p = 0, \ldots, P_{kn} - 1)$ are some canonical systems of eigenvectors and associated vectors of the matrix $B_{\alpha\beta}(\gamma)$ corresponding to γ_k and, respectively, of the conjugate matrix $\overline{B_{\beta\alpha}(\gamma_k)}$ corresponding to $\overline{\gamma}_k$, $D_{\alpha\beta}(\gamma)$ is a matrix function holomorphic at γ_k. From point 3 of Section 1 in [22], it follows also that $\sum_{n=1}^{N_k} P_{kn} = N_k^\circ$; in other words, the algebraic multiplicity of the eigenvalue γ_k for the matrix $B_{\alpha\beta}(\gamma)$ at $\gamma = \gamma_k$ is equal to the multiplicity of the zero γ_k for the function $\Delta(\gamma)$.

For a strip S' in the γ-plane, let S'_r denote a perforated strip obtained from S' after deleting all circular $r-$neighborhoods of the zeros $\gamma_k \in \overline{S'}$. If $\overline{S'}$ does not include γ_k, then $S'_r = S'$. Suppose there is a constant $M^\vee$ for any S'_r and any $\tilde{\theta}_j^{(m)} \in (\theta_{j-}^{(m)}\theta_{j+}^{(m)})$ such that

$$|B_{4m-4+j,\beta}^{-1}(\gamma)| < M^\vee|e^{i\gamma\theta}|\ \forall\ (\gamma,\theta) \in \overline{S'_r} \times [\theta_{j-}^{(m)}, \theta_{j+}^{(m)}], \tag{3.7}$$

$$|\tilde{D}^{(m)}_{j\beta-}(\gamma)| < M^{\vee}|e^{i\gamma\theta}| \ \forall\ (\gamma,\theta) \in \overline{S'_r} \times [\theta^{(m)}_{j-}, \tilde{\theta}^{(m)}_{j}], \tag{3.8}$$

$$|\tilde{D}^{(m)}_{j\beta+}(\gamma)| < M^{\vee}|e^{i\gamma\theta}| \ \forall\ (\gamma,\theta) \in \overline{S'_r} \times [\tilde{\theta}^{(m)}_{j}, \theta^{(m)}_{j+}]. \tag{3.9}$$

If a strip S does not contain γ_k, then (3.7)–(3.9) hold in $\overline{S'_r} = \overline{S'}$ for any closed strip $\overline{S'} \subset S$. Particularly, it takes place for any $\overline{S'} \subset S(\delta_+, 1)$, where $\delta_+ := \max_{\Re\gamma_k<1}(\Re\gamma_k)$.

For the V-junction, properties (3.7)–(3.9) can be obtained by a direct analysis of the matrix (3.3); for other junctions that were considered, it holds too.

Let the class $H^\circ_2(S(\delta_0, \delta_\infty))$ consist of functions $\Phi^\circ(\gamma)$ holomorphic in a strip $S(\delta_0, \delta_\infty)$ and such that the norm $[\int_{-\infty}^{\infty} |\Phi^\circ(\delta + i\xi)|^2 d\xi]^{1/2}$ is uniformly bounded with respect to δ on any segment $[\delta'_0, \delta'_\infty] \subset (\delta_0, \delta_\infty)$.

Let us denote $\delta_{g+} := \max(\delta_g, \delta_+)$. Owing to (2.11), we get from Theorem 1.7 in [18] that $\mathcal{G}_\alpha$ belong to $H^\circ_2(S(\delta_g, 1))$. Hence, taking into account properties (3.7)–(3.9), we have from Lemma 1.14 in [18] that the functions $\Phi^{(m)\vee}_j$, $\Psi^{(m)\vee}_j$ given by (3.5) belong to $\tilde{H}^\vee_2(\theta^{(m)}_{j-}, \theta^{(m)}_{j+}; S(\delta_{g+}, 1))$. Then it follows from Theorem 1.15 in [18] that after the inverse Mellin transform we get the solutions

$$\begin{aligned}
\Phi^{(m)}_j(z_j) &:= \int_{\delta-i\infty}^{\delta+i\infty} \Phi^{(m)\vee}_j(\gamma) z_j^{-\gamma}\, d\gamma, \\
\Psi^{(m)}_j(z_j) &:= \int_{\delta-i\infty}^{\delta+i\infty} \Psi^{(m)\vee}_j(\gamma) z_j^{-\gamma} d\gamma, \quad \delta \in (\delta_{g+}, 1)
\end{aligned} \tag{3.10}$$

that meet the *a priori* condition (2.12) for $\tilde{\delta}_0 = \delta_{g+}$. Thus the solution that we looked for is obtained and it is unique.

4. Stress asymptotics

Let us investigate now the stress asymptotics as $\rho \to 0$. Let $\delta_- := \min_{\Re\gamma_k>\delta_g}(\Re\gamma_k)$. It follows from the membership $\mathcal{G}_\alpha \in H^\circ_2(S(\delta_g, 1))$ together with Lemma 1.6 and Remark 1.5 in [18], that $\mathcal{G}_\alpha(\gamma)$ is uniformly bounded in any closed strip $\overline{S'} \subset S(\delta_g, 1)$. Then, taking into account the estimates (3.7)–(3.9), we may shift, as usual (see *e.g.* [1]), the integration path in (3.10) to the left into the strip $S(\delta_g, \delta_-)$, calculating the residues of the integrands at zeros γ_k of the function $\Delta(\gamma)$ in the strip $\overline{S}(\delta_-, \delta+)$. Thus, using representations (3.6) for the residue calculations, we get the asymptotics for the Kolosov–Muskhelishvili potentials

$$\Phi^{(m)}_j(z_j) = \sum_{\delta_g<\Re\gamma_k<1} z_j^{-\gamma_k} \sum_{n=1}^{N_k} \sum_{p=0}^{P_{kn}-1} K_{knp} \sum_{q=0}^{p} \frac{1}{q!} \log^q\left(\frac{1}{z_j}\right) \phi^{(n,p-q)}_{4m-4+j,k} + \Phi^{(m)}_{*j}(z_j), \tag{4.1}$$

$$\Psi^{(m)}_j(z_j) = \sum_{\delta_g<\Re\gamma_k<1} z_j^{-\gamma_k} \sum_{n=1}^{N_k} \sum_{p=0}^{P_{kn}-1} K_{knp} \sum_{q=0}^{p} \frac{1}{q!} \log^q\left(\frac{1}{z_j}\right) \phi^{(n,p-q)}_{4m-2+j,k} + \Psi^{(m)}_{*j}(z_j), \tag{4.2}$$

$$K_{knp} = \sum_{v=0}^{P_{kn}-p-1} \sum_{\beta=1}^{4M^*} \chi^{(n,P_{kn}-v-p-1)}_{\beta k} \mathcal{G}^{(v)}_{\beta k}, \qquad \mathcal{G}^{(v)}_{\beta k} := \frac{1}{v!}\frac{d^v}{d\gamma^v}\mathcal{G}_\beta(\gamma)|_{\gamma=\gamma_k}. \tag{4.3}$$

The remainder terms $\Phi_{*j}^{(m)}(z_j)$, $\Psi_{*j}^{(m)}(z_j)$ have the form (3.10) for $\delta \in (\delta_g, \delta_-)$ and belong to $H_2(\delta_g, \delta_-; W_j^{(m)})$ (see Theorem 1.15 in [18]). Hence by Lemma 1.10 in [18], for any $\overline{W_j'} \subset W_j^{(m)}$ and any $[\delta_0', \delta_\infty'] \subset (\delta_g, \delta_-)$ there is a parameter $\tilde{M}(\tilde{\theta}_{j-}^{(m)}, \tilde{\theta}_{j+}^{(m)}; \delta_0', \delta_\infty') < \infty$ such that

$$|\Phi_{*j}^{(m)}(z_j)|, |\Psi_{*j}^{(m)}(z_j)| \leqslant \tilde{M}|z_j|^{-\delta}, \quad \{z_j, \delta\} \in \overline{W_j'} \times [\delta_0', \delta_\infty']. \tag{4.4}$$

Substituting then (4.1)–(4.4) in (2.10), we obtain the stress asymptotics

$$\begin{aligned}\sigma_{\alpha\beta}^{(m)}(\rho,\theta) &= \sum_{\delta_g<\Re\gamma_k<1} \rho^{-\gamma_k} \sum_{n=1}^{N_k} \sum_{p=0}^{P_{kn}-1} K_{knp} \sum_{q=0}^{p} \log^q\left(\frac{1}{\rho}\right) F_{\alpha\beta kn,p-q}^{(m)}(\theta) + \sigma_{*\alpha\beta}^{(m)}(\rho,\theta),\\ &\qquad |\sigma_{*\alpha\beta}^{(m)}(\rho,\theta)| < M_*\rho^{-\delta_g-\epsilon},\\ &\qquad \forall\,\varepsilon \in (0, \delta_- - \delta_g),\ \{\rho,\theta\} \in \overline{W'} \subset W^{(m)},\ M_*(\varepsilon; \overline{W'}) < \infty.\end{aligned} \tag{4.5}$$

The parameters N_k and P_{kn} in (4.1)–(4.5) were presented above. The stress intensity factors K_{knp} depend on the right-hand sides of the boundary conditions, and they are explicitly expressed by (4.3) for a junction of infinite wedge-shaped plates (for a more arbitrary plate geometry such dependence is more complicated and usually is obtained by solving the complete boundary-value problem). For each γ_k, the number of stress intensity factors K_{knp} is equal to $\sum_{n=1}^{N_k} P_{kn} = N_k^\circ$, *i.e.*, to the multiplicity of the zero of the determinant $\Delta(\gamma)$. The functions $F_{\alpha\beta kn,p-q}^{(m)}(\theta)$ are infinitely smooth and can be written explicitly. For example,

$$\begin{aligned}F_{\rho\rho kn,p-q}^{(m)}(\theta) &:= \frac{1}{2}\sum_{j=1}^{2} \mathrm{e}^{-\mathrm{i}\gamma_k\theta_j} \sum_{w=0}^{p-q} \frac{1}{(p-q-w)!}(-\mathrm{i}\theta_j)^{p-q-w}\\ &\qquad \times[(2+\gamma_k)\phi_{4m-4+j,k}^{(nw)} + (1-\delta_{0w})\phi_{4m-4+j,k}^{(n,w-1)} - \mathrm{e}^{2\mathrm{i}\theta_j}\phi_{4m-2+j,k}^{(nw)}]\end{aligned}$$

for $\sigma_{\rho\rho}^{(m)}$, where δ_{0w} is the Kronecker delta and the functions $\phi_{\alpha k}^{(nw)}$ were described above.

One can see from (4.5) that if $\delta_g < 0$, particularly, if $g_i(\rho)$, $\mathrm{d}f_i(\rho)/\mathrm{d}\rho = \mathcal{O}(\rho^\epsilon)$, $\varepsilon > 0$, as $\rho \to 0$, then the stress singularities are determined by the zeros γ_k of the determinant $\Delta(\gamma)$ of the matrix $B_{\alpha\beta}(\gamma)$ in the strip $0 \leqslant \Re\gamma < 1$. When there are no zeros there, stresses and strains are bounded (in any wedge internal to $W^{(m)}$), and they may be singular in the opposite case.

So far, we considered the problem for an infinite wedge-shaped plate junction. By means of the cut-off function technique, one can prove analogous to [1–4], that the asymptotic representation (4.5) holds also for junctions of arbitrarily shaped plates, having the same wedge-shaped local geometry near the singular point. The only difference in comparison with the infinite wedge-shaped plates case is that the stress intensity factors K_{knp} can not be calculated explicitly by formula (4.3) and depend on loads in a more complicated manner. Let us give a sketch of the proof.

Let a solution to the problem for an arbitrary junction exists such that the displacements $u_i^{(m)}$ belong to the Sobolev space $\mathcal{W}_1^2$ in any finite domain, *i.e.*, the solution possesses a finite elastic energy there. Let us choose a sufficiently small radius $R > 0$ near the singular point such that the plate edges are straight and the boundary loads $g_j(\rho)$, $\mathrm{d}f_j(\rho)/\mathrm{d}\rho$ belong to $\hat{L}_2(\delta_g, 1)$, $\delta_g < 1$, on the interval $0 < \rho < R$. Let $\eta(\rho) \in C^\infty$ be a cut-off function such

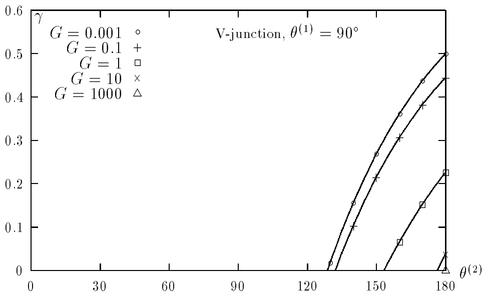

Figure 2. Dependence of γ_k on $\theta^{(2)}$ at $\theta^{(1)} = 90°$ for different G in the V-shaped junction.

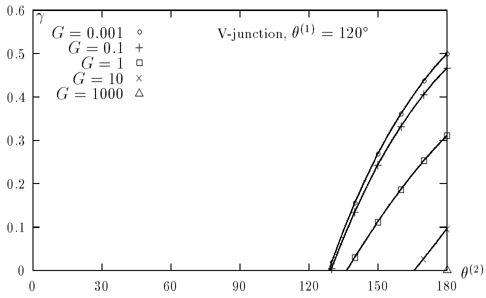

Figure 3. Dependence of γ_k on $\theta^{(2)}$ at $\theta^{(1)} = 120°$ for different G in the V-shaped junction.

that $\eta(\rho) = \{1, 0 \leqslant \rho \leqslant R/2;\ \ 0, \rho > R\}$. Then the functions $\eta u_i^{(m)}$ present a solution of the auxiliary problem for the Lamé system in the corresponding infinite junction: with mass forces $f_i^{(m)}$ being nonzero only at $R/2 \leqslant \rho \leqslant R$; with right-hand sides of the boundary conditions being nonzero at $0 < \rho \leqslant R$, differing from the initial ones only at $R/2 \leqslant \rho \leqslant R$, and belonging to $\hat{L}_2(\delta_g, \infty)$. This solution belongs to $\mathcal{W}_1^2$ in the whole of the wedges $W^{(m)}$.

We look for a solution of the same problem for the infinite junction in the form $v_i^{(m)} = \tilde{v}_i^{(m)} + V_i^{(m)}$. Here $V_i^{(m)}$ is the volume potential (for $f_i^{(m)}$), whose kernel is the Green function of the problem for the half plane that the boundary is directed along one of the edges of $W^{(m)}$ with the zero displacements at this boundary. Then $\tilde{v}_i^{(m)},\ m = 1, \ldots, M^*$, satisfy the homogeneous Lamé system and the boundary conditions with right-hand sides $\tilde{g}_j(\rho)$, $\mathrm{d}\tilde{f}_j(\rho)/\mathrm{d}\rho \in \hat{L}_2(\delta_{g0}, 2)$, where $\delta_{g0} := \max(\delta_g, 0)$. Solving the problem for $\tilde{v}_i^{(m)}$ as described in the previous sections, we obtain the solution with asymptotics (4.5) and such that its complex potentials belong not only to $\tilde{H}_2(\delta_{g0+}, 1;\ W_j^{(m)})$, but also to $\tilde{H}_2(\delta_{g0+}, \delta_{1+};\ W_j^{(m)}) \subset \tilde{H}_2(\delta_{g0+}, 1;\ W_j^{(m)})$, where $\delta_{g0+} := \max(\delta_{g+}, 0)$, $\delta_{1+} := \min_{\Re\gamma_k > 1}(\Re\gamma_k)$. The last membership holds, since either the point $\gamma = 1$ is not a zero of $\Delta(\gamma)$ or it is a zero, but $K_{1np} = 0$ for $\gamma_1 = 1$ (what follows from satisfying the solvability conditions for both the original problem for $u_i^{(m)}$ and the auxiliary problem for $\tilde{v}_i^{(m)}$). It follows from that membership (see Lemmas 1.17, 1.18 in [18]) that $\tilde{v}_i^{(m)}$ belong to the Sobolev space $\mathcal{W}_1^2$ in the whole of the infinite wedges $W^{(m)}$. The volume potential and, consequently, $v_i^{(m)}$ have the same property.

Since the uniqueness (in stresses) takes place in this space, the solutions $\eta u_i^{(m)}$ (which also belong to $\mathcal{W}_1^2$) and $v_i^{(m)}$ may differ only by a rigid-body displacement. Thus, the stress asymptotics for an arbitrary and the corresponding infinite junctions coincide at least on the singular interval $0 < \gamma_k < 1$. Considering more carefully the asymptotics of the volume potential $V_i^{(m)}$ and of its boundary traces as $\rho \to 0$, it is possible to show that the asymptotics coincide for the whole interval $\delta_g < \gamma_k < 1$.

The plate bending rigidity will change, generally speaking, the stress asymptotics, but this change will concern only a small region surrounding the singular point, having a dimension of the order of the plate thickness. At a distance that is large with respect to the thickness, the membrane asymptotics presented here holds.

By use of the methods of [11–12], the investigation techniques given in this paper can be extended to the study of the stress singularities in thin intersecting anisotropic plates. Applying the methods of [14–15] one can investigate the stress singularities also in thin hereditary-elastic (visco-elastic) plate junctions.

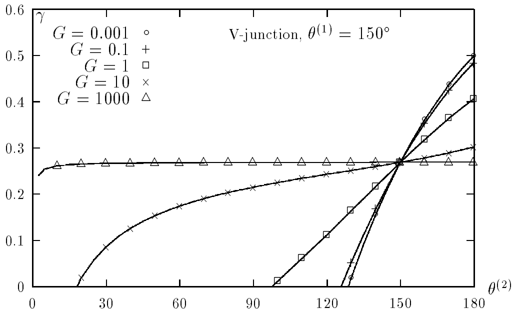

Figure 4. Dependence of γ_k on $\theta^{(2)}$ at $\theta^{(1)} = 150°$ for different G in the V-shaped junction.

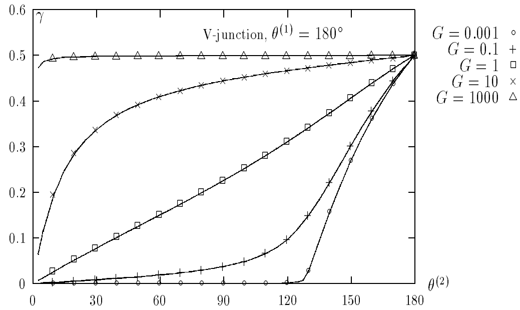

Figure 5. Dependence of γ_k on $\theta^{(2)}$ at $\theta^{(1)} = 180°$ for different G in the V-shaped junction.

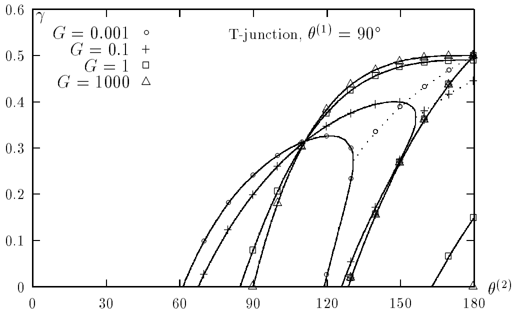

Figure 6. Dependence of γ_k on $\theta^{(2)}$ at $\theta^{(1)} = 90°$ for different G in the T-shaped junction.

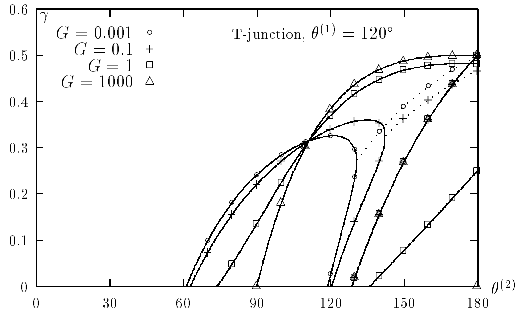

Figure 7. Dependence of γ_k on $\theta^{(2)}$ at $\theta^{(1)} = 120°$ for different G in the T-shaped junction.

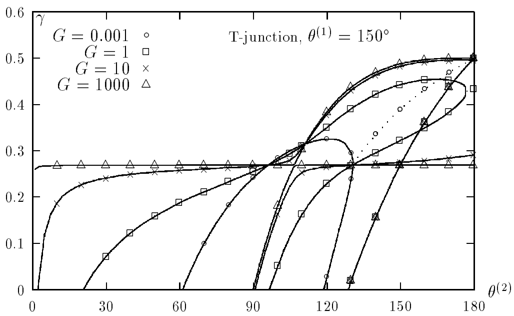

Figure 8. Dependence of γ_k on $\theta^{(2)}$ at $\theta^{(1)} = 150°$ for different G in the T-shaped junction.

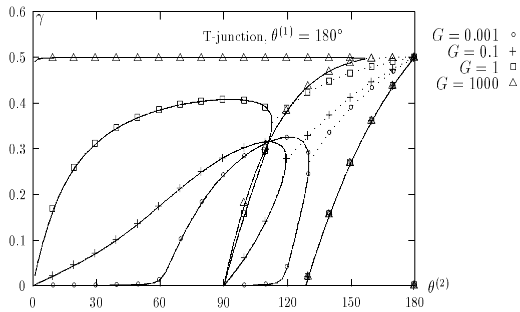

Figure 9. Dependence of γ_k on $\theta^{(2)}$ at $\theta^{(1)} = 180°$ for different G in the T-shaped junction.

5. Numerical examples

The singularity powers γ_k, that is, the zeros of the determinant $\Delta(\gamma)$, were evaluated in the strip $S(0, 1)$ for different plate junctions by the Müller (complex parabola) method, which calculates real and complex roots of a complex function. The numerical results for the V- and T-shaped junctions are given in Figures 2–9, representing the dependence of γ_k on the angle $\theta^{(2)}$ at some fixed angles $\theta^{(1)}$ at different values of the rigidity parameter $G = G^{(21)} = (H^{(2)}\mu^{(2)})/(H^{(1)}\mu^{(1)})$ for $\nu^{(1)} = \nu^{(2)} = 0{\cdot}3$. (The case $\theta^{(3)} = \theta^{(2)}$, $H^{(3)}\mu^{(3)} = H^{(2)}\mu^{(2)}$, $\nu^{(3)} = \nu^{(2)}$ is presented for the T-junction).

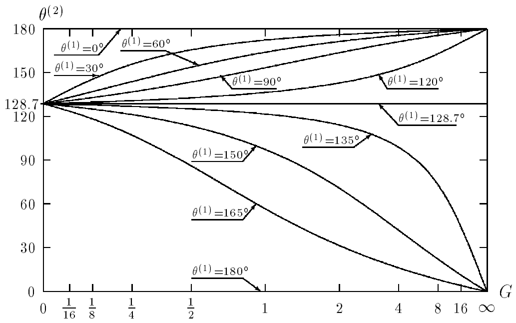

Figure 10. Dependence of $\theta^{(2)}$ on G for different θ^1 in the V-shaped junction.

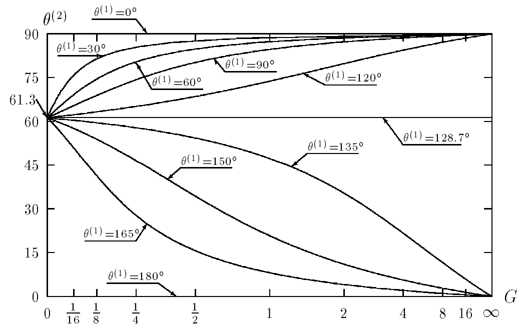

Figure 11. Dependence of $\theta^{(2)\star} = \theta^{(3)\star}$ on G for different $\theta^{(1)}$ in the T-shaped junction.

There exist only real roots γ_k in the strip $0 < \Re\gamma < 1$ for the analysed V-shaped junctions. For the T-shaped junctions, complex roots occur for some considered parameters; the real parts of these roots are presented for such cases by dotted lines on the figures.

The case $G = 0{\cdot}001$ coincides with the limiting case $G \to 0$ almost at all points. The singularity powers for this limiting case correspond to the union of the singularity powers for two separate problems. For the V-shaped junction, the first is the problem for the $W^{(1)}$ plate with some tractions prescribed on both boundaries $\Gamma^{(1)}$ and $\Gamma^{(3)}$ (it generates no singularities for the parameters considered), the second is the problem for the $W^{(2)}$ plate with some tractions prescribed on the boundary $\Gamma^{(2)}$ and some normal traction and tangent displacement prescribed on the boundary $\Gamma^{(3)}$ (the singularity powers are independent of $\theta^{(1)}$). For the T-shaped junction, the first is also the problem for the $W^{(1)}$ plate with some tractions prescribed on both boundaries $\Gamma^{(1)}$ and $\Gamma^{(4)}$ (it generates no singularities for the parameters considered), the second is the problem for the joined $W^{(2)} \cup W^{(3)}$ plate with some tractions prescribed on both boundaries $\Gamma^{(2)}$ and $\Gamma^{(3)}$ and, in addition, with some tangent displacement and with the continuity conditions for the normal traction and normal displacement prescribed on the line $\Gamma^{(4)}$, *i.e.* with the conditions of thin inextensible fiber on this line, (the singularity powers are independent of $\theta^{(1)}$).

The case $G = 1000$ coincides with the limiting case $G \to \infty$ almost at all points. The singularity powers for this limiting case correspond to the union of the singularity powers for two separate problems. For the V-shaped junction, the first is the problem for the $W^{(1)}$ plate with some tractions prescribed on the boundary $\Gamma^{(1)}$ and some normal traction and tangent displacement prescribed on the boundary $\Gamma^{(3)}$ (the singularity powers are independent of $\theta^{(2)}$), the second is the problem for the $W^{(2)}$ plate with some tractions prescribed on both boundaries $\Gamma^{(2)}$ and $\Gamma^{(3)}$ (it generates no singularities for the parameters considered). For the T-shaped junction, the first is also the problem for the $W^{(1)}$ plate with some tractions prescribed on the boundary $\Gamma^{(1)}$ and some normal traction and tangent displacement prescribed on the boundary $\Gamma^{(4)}$ (the singularity powers are independent of $\theta^{(2)}$), the second is the problem for the joined $W^{(2)} \cup W^{(3)}$ plate with some tractions prescribed on both boundaries $\Gamma^{(2)}$ and $\Gamma^{(3)}$ (the singularity powers are independent of $\theta^{(1)}$).

There is also a curve for the T-shaped junction that is independent of the rigidity parameter G and the angle $\theta^{(1)}$. It corresponds to the singularity powers generated by the antisymmetric deformation mode for the joined $W^{(2)} \cup W^{(3)}$ plate under the action of some antisymmetric tractions prescribed on both boundaries $\Gamma^{(2)}$ and $\Gamma^{(3)}$.

One can see from these pictures that there are critical values $\theta^{(2)\star}$ of the angle $\theta^{(2)}$ that are boundaries between the parametric zones with and without the singularities $\gamma_k > 0$. It is possible to determine the dependence of $\theta^{(2)\star}$ on G. These dependencies are presented in Figures 10 and 11 for the V- and T-junctions at some fixed angles $\theta^{(1)}$. To show the half-infinite interval $0 < G < \infty$ in the figures, the mapping $\tilde{G} = 2G/(1+G)$ was used, and the G-axis is linear with respect to $\tilde{G}$. If a point $\{G, \theta^{(2)}\}$ lies above the corresponding critical curve for an angle $\theta^{(1)}$, then a singularity $\gamma_k > 0$ arises at these parameters. If this point lies below the curve, the stress singularity is absent.

Note that for the V-junction, $\theta^{(2)\star} \to \theta^{\star}_{V0}$ as $G \to 0$ for all $\theta^{(1)}$. The angle $\theta^{\star}_{V0} \approx 128{\cdot}7^{\circ}$ is the solution of the equation $2\theta = \tan(2\theta)$ and is the corresponding critical value for the plate $W^{(2)}$ with some tractions prescribed on the boundary $\Gamma^{(2)}$ and some normal traction and tangent displacement prescribed on the boundary $\Gamma^{(3)}$. For $\theta^{(1)} > \theta^{\star}_{V0}$, the critical angle $\theta^{(2)\star}$ increases monotonically with growing G, reaching 180° at $G = \infty$. For $\theta^{(1)} < \theta^{\star}_{V0}$, the critical angle $\theta^{(2)\star}$ decreases monotonically with growing G, reaching zero at $G = \infty$.

For the T-junction, $\theta^{(2)\star} \to \theta^{\star}_{T0}$ as $G \to 0$ for all $\theta^{(1)}$. The angle $\theta^{\star}_{T0} = \arcsin(\sqrt{(1+\kappa)/4}) \approx 61{\cdot}3^{\circ}$ is the corresponding critical value for the joined plate $W^{(2)} \cup W^{(3)}$ with some tractions prescribed on both boundaries $\Gamma^{(2)}$ and $\Gamma^{(3)}$ and with the conditions of thin inextensible fiber prescribed on the line $\Gamma^{(4)}$. For $\theta^{(1)} > \theta^{\star}_{V0}$, the critical angle $\theta^{(2)\star}$ increases monotonically with growing G reaching 90° at $G = \infty$. For $\theta^{(1)} < \theta^{\star}_{V0}$, the critical angle $\theta^{(2)\star}$ decreases monotonically with growing G, reaching zero at $G = \infty$.

Thus, analysing these curves, one can give some recommendations concerning plate junction optimization to avoid the stress singularities in singular points.

6. Conclusion

Our analysis has shown:

(1) The problem for a space junction of thin elastic wedge-shaped plates can be reduced to a boundary-value problem for a system of partial differential equations of plane elasticity with non-traditional transmission conditions. A general solution of this system can be expressed in terms of the Kolosov–Muskhelishvili complex potentials. This reduces the problem to a boundary-value/transmission problem for holomorphic functions belonging to corresponding weighted Hardy-type classes.

(2) Application of the Mellin transforms and the classes properties allows to obtain singular stress asymptotics near the joint apex. The stress singularity powers (exponents) are the zeros of an explicitly written determinant, whose order equals twice the number of plates involved in the junction. It is pointed out that the results of this analysis hold also for more arbitrary junctions.

(3) The numerical examples of the stress singularity powers dependence on elastic parameters and geometry for the V- and T-junctions are given. They show the feasibility of the analysis and allow to calculate the stress-singularity powers to use them in general numerical methods, *e.g.*, by introducing special singular elements in the Finite-Element or the Boundary-Element Method. Another possible application of the results is a special choice of the junction elastic and/or geometrical parameters to avoid the stress singularity in the junction model and the stress concentration in a real plate junction.

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